ECE-4632, Midterm exam — François Meyer

Open notes, and official class textbook

Answer all problems

Problem 1

Let $h[n]$ be the impulse response of a linear time invariant system defined by

$$h[n] = \begin{cases} a^{n-1}\cos(\omega_0 n) & \text{if } n \geqslant 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Find the difference equation that characterizes the system. (Hint : you may consider computing the z-transform of the impulse response h).

Problem 2

We are given the filter

$$H(z) = \frac{z^{-1} + 0.5}{1 + 0.5z^{-1}} \tag{2}$$

and the two input signals

$$x_1[n] = \sin(\frac{\pi}{10}n), \quad x_2[n] = 3\cos(\frac{4\pi}{10}n) \tag{3}$$

Let $y_1[n]$ and $y_2[n]$ be the output signals corresponding to $x_1[n]$ and $x_2[n]$.

1. Find $y_1[n]$ and $y_2[n]$? Answer with the minimum amount of computations.

Problem 3

Consider the causal system defined by the pole-zero plot shown in Fig. 1 (note the pole at 0).

1. Determine the z-transform of the system given that $H(z)|_{z=1} = 1$.
2. Find the impulse response $h[n]$.
3. Is the system causal ?

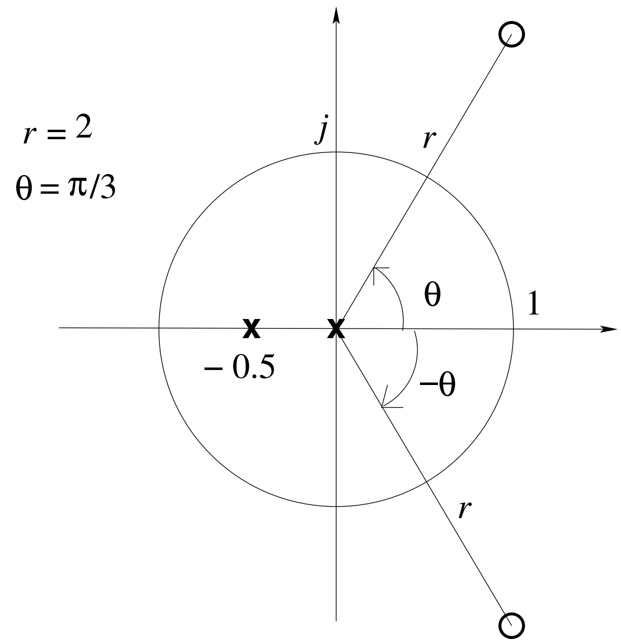

Figure 1: Pole-zero plot. Note the pole at 0.

Problem 4

The continuous time signal $x_c(t)$ is band limited, and has the Fourier transform

$$X_c(j\Omega) = \begin{cases} \cos^2(\Omega/4) & \text{if } |\Omega| \leqslant 2\pi, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The signal $x_c(t)$ is sampled with a period $T = 1$ to create the sampled signal $x_s(t)$.

1. Find the expression of the Fourier transform $X_s(j\Omega)$ of the sampled signal $x_s(t)$.
   Hint: since $X_s(j\Omega)$ is 2π periodic, you can calculate $X_s(j\Omega)$ for $\Omega \in [0, 2\pi]$, and use the fact that $\cos(\alpha - \pi/2) = \sin(\alpha)$.

2. You know from the sampling theorem that the Fourier transform $X(e^{j\omega})$ of the discrete sequence $x[n] = x_c(nT)$ is given by

   $$X(e^{j\omega}) = X_s(j\frac{\omega}{T}) \tag{5}$$

   where $T = 1$. Find the sequence $x[n]$.